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Class: -X **Subject: -Mathematics**

Linear Equation in Two Variables

Basic Concepts with Examples

Linear Equation in Two Variables: -**An equation in the form ax + by + c = 0, where a, b and c are real numbers {a ≠ 0, or b≠ 0) is called a linear equation in two variables 'x' and 'y'.**

Example 2 x + 3 y = 12

It has infinite many solutions **(You have already studied in class IX**th**)**

x	0	6	3	…
y	4	0	2	…
(x, y)	(0, 4)	(6, 0)	(3, 2y)	(…, …)

Simultaneous linear equations: **Two linear equations in two variables taken together are called simultaneous linear equations. The solution of system of simultaneous linear equation is the ordered pair (x, y) which satisfies both the linear equations.**

General Form of a Pair of Linear Equations in Two Variables

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where a_1 b_1, c_1, a_2, b_2, c_2 are real numbers such that $a_1^2 + b_1^2 = 0$ and $a_2^2 + b_2^2 = 0$

Ratio of x = $\frac{a_1}{a_2}$

Ratio of y = $\frac{b_1}{b_2}$

Ratio of constant = $\frac{c_1}{c_2}$

Example:

$$2x + 3y = 12$$

$$5x + 6y = 18$$

Given: $a_1 = 2$, $a_2 = 5$, $b_1 = 3$, $b_2 = 6$, $c_1 = 12$ & $c_2 = 18$

Ratio of x = $\frac{a_1}{a_2} = \frac{2}{5}$,

Ratio of y = $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$,

Ratio of constant = $\frac{c_1}{c_2} = \frac{12}{18} = \frac{2}{3}$,

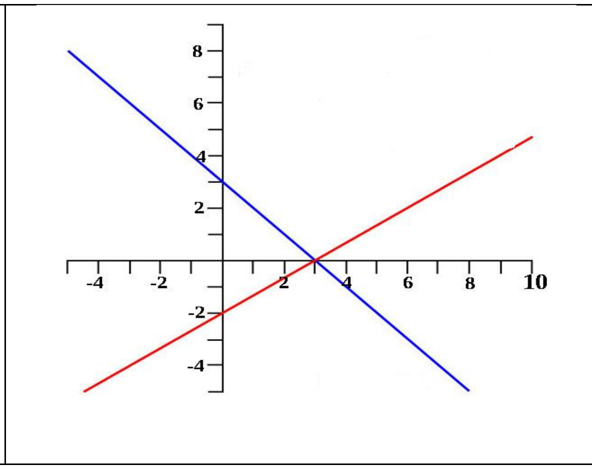
There are three situations can arise:

(1) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ or $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ *or* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(a) The system is consistent

(b) It has only one solution or unique solution &

(c) The graphs of two equations of a system intersect at a point (intersecting lines)



Example: $2x + 3y = 12$

$5x + 6y = 18$

Given: $a_1 = 2$, $a_2 = 5$, $b_1 = 3$, $b_2 = 6$, $c_1 = 12$ & $c_2 = 18$

Ratio of x = $\frac{a_1}{a_2} = \frac{2}{5}$,

Ratio of y = $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$,

Ratio of constant = $\frac{c_1}{c_2} = \frac{12}{18} = \frac{2}{3}$,

$\Rightarrow \frac{2}{5} \neq \frac{1}{2} \neq \frac{2}{3}$ ⍰ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

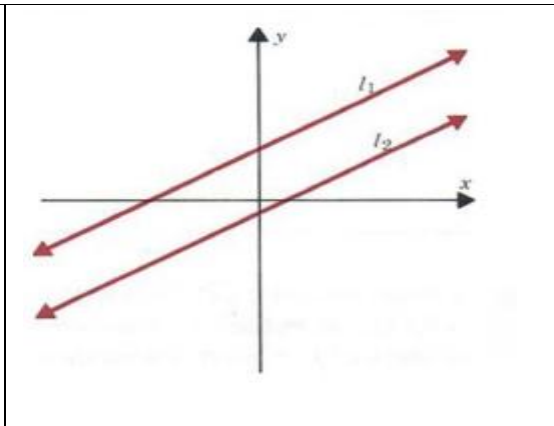
Hence: If the graphs of two equations of a system intersect at a point, the system is said to have a unique solution, i.e., the system is consistent.

(1) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

(a) The system is inconsistent

(b) It has no solution &

(c) The graphs of two equations of a system are two parallel lines.



Example: $2x + 3y = 12$

$4x + 6y = 18$

Given: $a_1 = 2$, $a_2 = 4$, $b_1 = 3$, $b_2 = 6$, $c_1 = 12$ & $c_2 = 18$

Ratio of x = $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

Ratio of y = $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

Ratio of constant = $\frac{c_1}{c_2} = \frac{12}{18} = \frac{2}{3}$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} \neq \frac{2}{3} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

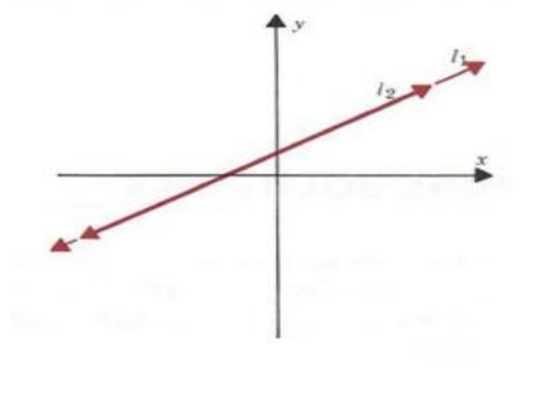
Hence: If the graphs of two equations of a system are two parallel lines, the system is said to have no solution, i.e., the system is inconsistent.

(2) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(a) The system is consistent and dependent

(b) It has infinitely many solutions &

(c) The graphs of two equations of a system are two coincident line



Example: $2x + 3y = 12$

$4x + 6y = 24$

Given: $a_1 = 2$, $a_2 = 4$, $b_1 = 3$, $b_2 = 6$, $c_1 = 12$ & $c_2 = 24$

Ratio of x = $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

Ratio of y = $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

Ratio of constant = $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence: If the graphs of two equations of a system are two coincident lines, the system is said to have infinitely many solutions, i.e., the system is consistent and dependent.

DO YOUR SELF

Identify given equations based on the conditions or situations

(a)	(b)	(c)	(d)
$2x + 5y = 16$	$x - 2y = 4$	$5x - 6y = 10$	$5x - 6y = 10$
$3x - 6y = 24$	$3x - 6y = 2$	$2.5x - 3y = 20$	$-3x + 6y = 2$